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AN ATTENTIONAL MODEL FOR THE CONCEPTUAL CONSTRUCTION OF UNITS AND NUMBER

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Just before the turn of the century, John Dewey wrote, "Number is a rational process, not a sense fact" (McLellan & Dewey, 1895/1908, p. 24). He put in a nutshell an idea that had been suggested, implied, or tacitly taken for granted by quite a few philosophers before him. His introduction of the term *process*, however, emphasized an aspect that had hitherto been largely disregarded. Berkeley, nearly 200 years earlier, had said that "number is entirely the creature of the mind" (1710/1963, p. 36). But since Berkeley referred to all products of sensation as "ideas," his readers have tended to interpret his statement about number as saying no more than what he said about almost everything else.

Kant, about halfway between Berkeley and Dewey, comes a good deal closer. Unity, plurality, and totality are the three categories of quantity for Kant, and they arise from "pure synthesis" of the a priori conditions of all experience, space, and time. Though Kant says that we are rarely conscious of the workings of this synthesis, it does appear to be a constructive activity or process in that the "pure" categories, including those of quantity, are in some way compiled out of the conceptions of space and time as the original, a priori raw material (Kant, 1787/1897, p. 45ff). The question as to what should be assumed as raw material is the point where Jean Piaget, in many ways himself a Kantian, disagrees. In his genetic epistemology, the concepts of space and time are no more a priori than those of existence, causality, or number. All of them are for Piaget the constructions of an active mind that conceptually organizes both itself and its world by crystallizing experience into an interaction of an experiencing subject and the objects of experience (Piaget, 1937, p. 311ff).

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I wish to acknowledge that Leslie P. Steffe's investigations and microanalyses of children's counting behavior and John Richard's examination of the philosophical roots and implications of our joint endeavor have been a constant stimulation and corrective in the development of the model I am presenting here. Our intensive interaction has led to many refinements. I also thank Gérald Noelting and an anonymous reviewer for helpful suggestions concerning an earlier version of this paper. Whatever misconceptions remain, however, are entirely mine.

There is no doubt that Piaget conceives the mind—the agent of all construction—as a rational one, and he would readily agree with Dewey that number is a rational process. Neither he nor Dewey, however, are successful in explicating, in terms of specific operations, what that process might be and how it comes to constitute number. Although both men are well aware of the importance of the concept of unity, neither ventures a hypothesis as to how that concept might be constructed. Dewey does expand on the statement previously quoted and says that number arises "from certain rational processes in construing, in defining and relating the material of sense perception" (McLellan & Dewey, 1895/1908, p. 35) and that definite ideas of number can be formed only from "the child's own activity in conceiving a whole of parts and relating parts in a definite whole" (p. 30–31). That is an excellent description of what has to be done, but it gives us no clue as to the kind of mechanism that could do it.

Piaget, however, characterizes the concept of unit by a negative description: "Elements are stripped of their qualities and become arithmetic unities" (1970, p. 37). Intuitively we feel that this does tell us something important about oneness and, indeed, about numerical concepts. But, again, it does not tell us what those concepts consist of. If we take that definition literally, it immediately raises the problem that if we strip an element of *all* its qualities, we are left with a nonentity—a problem we know only too well from the desperate struggles, ever since Aristotle, to maintain an amorphous "matter" as a support for properties. The trouble is that, whereas we can all agree that, say, what we call *two* has no color, no figural shape, no texture, nor any other sensory properties, it nevertheless has something that makes it definitely different from *one* or *three* or any other number. Hence, it cannot be true that it has *no* properties. To say, then, that what distinguishes numbers from one another is an *abstract* property does not help either, because the meaning of "abstract" has so far never been defined in other than negative terms (i.e., not concrete or not sensory).

To be fair, however, it must be remembered that Piaget defines *number* as an "operatory group structure, without which there cannot be conservation of numeric totalities independent of their figural disposition" (Piaget & Szeminska, 1964, p. 9). In all his work on the development of number, Piaget focuses on a conceptual complex that involves class inclusion and order, and, like most of his predecessors, he takes the construction of units for granted. In this paper, on the contrary, I am not concerned with the "numerical" relations and potentialities of number concepts (numerosity, ordinality, commutativity, etc.) but exclusively with mental operations capable of building up conceptual structures that might *have* such relations and potentialities.

The fact that numerical concepts and other "creatures of the mind" are not accessible to observation does not prevent us from hypothesizing mechanisms that could account for them. In the pages that follow, I present the outline of a model that provides a new perspective on the conceptual foundations of mathematics and a new approach for the study of children's acquisition of numerical operations. I emphasize that I am concerned with the theoretical exposition of the model rather than with its use in developmental investigations. A subsequent paper will provide a discussion of the application and the usefulness of the model in the analysis of children's counting (Steffe, Richards, & von Glasersfeld, Note 1).

The idea that the structure of certain abstract concepts could be interpreted as patterns of attention was first proposed by Ceccato (1966) in the context of his pioneering work in conceptual semantics. In our recent research aimed at the formulation of a theoretical model for the development of the concepts underlying numerical operations, the original idea has been modified and expanded (von Glasersfeld, Note 2). Before I outline the basic features of that approach, I want to emphasize that "attention," in this context, has a special meaning. Attention is not to be understood as a state that can be extended over longish periods. Instead, I intend a pulselike succession of moments of attention, each one of which may or may not be "focused" on some neural event in the organism. By "focused" I intend no more than that an attentional pulse is made to coincide with some other signal (from the multitude that more or less continuously pervades the organism's nervous system) and thus allows it to be registered. An "unfocused" pulse is one that registers no content.

The suggestion of a pulsating attention or consciousness can be found already in Craik (1948) and in Pitts and McCulloch (1947). I suspect that the idea is indeed much older, because it would seem to arise quite naturally from remarks von Helmholtz (1866) made on the experiential fact that we can shift our attention at will from one part of the visual field to another without moving our eyes. This observation was further documented by Köhler (1947) and amply demonstrated in experiments with the "fixed retinal image" technique (Pritchard, Heron, & Hebb, 1960; Zinchenko & Vergiles, 1972). Similarly, investigators of auditory perception have long known that, within certain acoustic limits, attention can be deliberately focused on any one of several simultaneous signal sequences ("cocktail party effect"). This in no way contradicts experimentally established rules governing the differential salience of sensory input in the visual or auditory channels—it merely means that the focusing of attention can at times disregard the relative salience of available signals.

In the present context, these findings are important because first, they confirm the assumption that attention operates above, and independently of, sensation and can, therefore, function as an organizing principle; second, if attention can, indeed, shift from one place in the experiential field to another, it must have a means of regarding (focusing on) these places and of disregarding whatever lies in between. Attention conceived of as a sequence of discrete moments would achieve this in the simplest possible fashion.

With regard to the pulselike character of attention, neurophysiology has recently provided data that tend to make that assumption a good deal more plausible than it seemed earlier. Harter (1967) surveyed a variety of experimental work that shows intermittence in perceptual and conceptual processes. Few authors, however, agreed on the physiological origin or the function of the rhythmic phenomenon. "The various aspects of the problem of accounting for intermittency have still to be sorted out" (Kohlers, 1972, p. 148). But the idea has received new impetus through the work of Varela, Toro, John, and Schwartz (Note 3), who reported experimental evidence that two flashes of light in quick succession are perceived as sequential or simultaneous depending on whether or not they fall within the positive phase of the subject's alpha rhythm. Consequently, they suggested that this cortical rhythm may, indeed, have a function that would be aptly described as "perceptual framing."

Both the observation of the stimulus-independent mobility of attention and the empirical finding that a pulselike phenomenon in the cortex plays a decisive role in the determination of certain perceptual results encouraged me to pursue the investigation of an attentional model for the conceptual construction that generates units, pluralities, and lots.

The reasons why some form of intermittence seems a necessary requirement in any analysis of perceptual processes were summarized by Harter (1967):

> First, if the brain utilizes its finite number of components in the most efficient manner, it should not operate continuously by processing all sensory information at all points in time, but should operate discontinuously by taking successive samples of sensory information at different points in time.... Second, the brain must code or label incoming sense information in terms of its time of occurrence.... Such coding is necessary for events to be cognitively placed in proper temporal sequence and for the sensory, associative, and motor cortical functions to operate on a common time basis. (p. 47)

The most important of these considerations, from the point of view of concept formation, are those of the associative functions. We *do* divide our visual, auditory, and tactual fields of experience into separate parts which, in our cognitive organization, then become individual items or "things." That is to say, we quite successfully differentiate or "cut" things out of a background and perceive each one of them as an entity or whole. Any closer analysis of this process seems to lead to the realization that entities that can be recognized, moved about, and, above all, counted in a perceptual field cannot be established by mere sensory differentiation. Whether it is the table in front of us, trees we see through the window, or a bunch of keys we feel in our pocket, none of them could be perceived as a discrete item on the strength of sensory differences alone. Many of the things we perceive as unitary items comprise, within their unity, sensory differences that are as great as, or even greater than, those by which they may be distinguished from their background. In addition, we observe that we can create unitary items in the absence of sensory differences. For instance, when we look at a blank sheet of paper and see it as an upper and a lower part, these two parts are not only conceptually separated from each other but also conceived as individual entities that could be physically separated if we chose to cut the sheet (we know where to cut it before we start cutting).

One may conclude, on the basis of these and other related observations, that the creation of conceptual units is the result of operations which, in principle, are not dependent on sensory material, although such material may be indispensable at the beginning of their development. The hypothesis I am here proposing is that these unitizing operations consist in the differential distribution of focused and unfocused attentional pulses. A group of co-occurring sensory-motor signals becomes a "whole" or "thing" or "object" when an unbroken sequence of attentional pulses is focused on these signals and the sequence is framed or bounded by an unfocused pulse at both ends. The unfocused pulses provide closure and set the sequence of contiguous focused pulses apart from prior and subsequent attentional pulses.

Piaget has long maintained that the concept of "object" requires the coordination of sensory material from more than one source (e.g., Piaget, 1937, p. 76). In practice this would most often be a combination of visual and tactual signals plus the proprioceptive (kinesthetic) signals deriving from the movements involved in visual scanning and tactual exploration. One everyday example of this is a rainbow: Because it is an exclusively visual phenomenon that provides no correlates in other sensory modes and shifts location relative to the observer's motion, we are reluctant to consider it an "object" in the ordinary sense of that word. Note also that a single sensory signal can never suffice for the conceptual construction of the duration, or "permanence," that we consider an essential constituent of objects.

In the proposed breakdown, therefore, an experiential object would always be composed of several pulses focused on different sensory signals. The attentional pattern of such a *sensory-motor item* can be represented by a graphic notation:

1) $$0 \underset{a}{I} \underset{b}{I} \cdots \underset{n}{I} 0$$ Sensory-Motor Item

"0" indicates unfocused and "I" focused pulses; "a, b, . . . n" indicate that the sequence of focused pulses involves focus on a variety of sensory-motor signals.

In the forthcoming analysis of children's counting, the hypothesis will be substantiated that the child's awareness of the kinds of items being counted is one of the developmental features that changes progressively in the acquisition of numerical skills. Among other considerations it will be important to differentiate sensory-motor items according to the particular perceptual, proprioceptive, or representational material on which the child focuses. In the present context, however, I shall treat sensory-motor items generically (i.e., regardless of whether they are constituted by visual, auditory, kinesthetic, or other signals) and merely deal with the two aspects they all have in common.

The first of these aspects is that a sensory-motor experience, in order to be conceptualized as a "thing," must contain more than one focused impulse. That could be, for instance, one perceptual signal (e.g., color, texture) and one proprioceptive signal (e.g., from eye movement registered in orienting to the perceptual signal); pulses focused on such signals would, respectively, make the "thing" qualitatively discriminable and give it location in the experiential field.

The second aspect is that of boundedness. This is indispensable if the experience is to be conceptualized as a "thing." Though the sensory material registered by the focused pulses may be heterogeneous, the succession of pulses as a whole must be discriminable from a background. Background is, of course, a relative construct in the sense that anything in the experiential field can serve as ground, provided we discriminate something as figure. What matters in the present context is that the sensory-motor experience is unified into a whole by the fact that certain sensory signals are consecutively focused on, whereas at least one pulse preceding them, and one following, are not focused, regardless of whatever other signals may or may not be available. These unfocused pulses then constitute the equivalent of a blank space—that is, a space that is considered blank because no signals are registered in it.

At an early stage of conceptual development, the unifying attentional procedure seems to require the support of easily discriminable sensory signals. In the visual, auditory, and tactual fields, discrimination may be helped by contrast, unification by experiential proximity; in the kinesthetic field, the beginning and the end of a motor act constitute an inherent boundary to the sensation the act creates, and a rhythm or other form of regularity may serve to generate continuity. Children, as a rule, acquire a good many names of things during their second year. Every time they learn to apply a new name appropriately, they must first have isolated an experiential item that more or less approximates the adult conception of the object that bears that name. To isolate an item from the experiential background, according to my hypothesis, involves the attentional pattern that bounds, or sets apart, a sequence of focused pulses by means of an unfocused one at both ends. That pattern, therefore, must be highly recurrent—and to separate it conceptually from the varying sensory material with which it occurs is probably among the very first instances of generalizing, or "empirical" abstraction (Piaget, 1974). That is to say, the attentional pattern will be recognized as similar whenever it occurs, regardless of the particular sensory-motor signals that are registered in the focused pulses. Yet, although it does not matter what the content of the focused pulses is, they must have experiential content. This becomes clear once we realize that it is that generalized construct of a unified whole that becomes linked to the linguistic form of the unitary singular as opposed to the plural. (The particular name is, of course, tied to the sensory-motor content.)

The notation indicates merely the essential characteristics, that is, the boundary of unfocused pulses around a focus on some sequence of sensory signals (n) that could be specified and that is now represented by one focused pulse, because the contained sensory-motor material is irrelevant for the conception of unity or wholeness:

2) $$0 \overset{\mathrm{I}}{\underset{\mathrm{n}}{\cdot}} 0 \qquad \text{Unitary Item}$$

When toddlers, during their second year, name objects one after the other (psycholinguists have called that early activity *labeling*), they create a succession of heterogeneous sensory-motor items, each of which is a separate unitary whole, independent of all others, and therefore not an element in the kind of succession that could be conceived as a *plurality*. A plurality can be formed only if the sensory-motor items of the succession share a sensory feature that provides a basis for considering them equivalent in that respect. That is to say, the conceptual construct of plurality is related to classification. A plurality, of course, is not yet a class; but the individual items of a plurality, like those of a class, must have at least one feature in common that serves as a connection among them all. Again, this is reflected in the ordinary use of language where a plural necessarily refers to items, each one of which could be individually referred to by the singular form of the word. It is important to note, however, that a plurality (i.e., what is referred to by any plural noun) is not bounded and, therefore, is conceived neither as one specific succession of units nor as a closed composite that would correspond to one and only one number-name.

Hence, one must represent plurality as an open-ended string of unitary items in their minimal form and indicate that the one focused pulse in each element contains the same experiential material that is the common feature that connects them.

3) $$\ldots 0 \overset{\mathrm{I}}{\underset{\mathrm{a}}{\cdot}} 0 \quad 0 \overset{\mathrm{I}}{\underset{\mathrm{a}}{\cdot}} 0 \quad 0 \overset{\mathrm{I}}{\underset{\mathrm{a}}{\cdot}} 0 \ldots \qquad \text{Plurality}$$

If you live downtown and a window of your ground-floor apartment looks out on the sidewalk of a busy street, you may see an endless plurality of people passing by. If, then, you consider the men, women, and children you saw pass, say, between breakfast and lunch, you have a plurality of people that is framed between two extraneous events that can be seen as part of your experiential background. At that point, the plurality turns into a *collection*. The elements of that collection are still individually different sensory-motor items, but they all have the feature or features that allow you to call them "people," and on that basis they can form a closed group against an experiential background.

Initially, collections are most likely to be formed in the visual field—a handful of cherries on a plate, a pile of books on a table, a group of trees against the sky. What matters is that a succession of sensory-motor items can be isolated from a background, that the items are in some respect the same and can, therefore, be seen as a plurality, and that the plurality is itself experienced against a background that contains it, in the sense that there is a surrounding area where no items of the particular kind can be isolated.

In the notation, this experiential boundedness is represented by means of parentheses.

4) $$\left(0 \overset{I}{\underset{a}{\,}} 0 \quad 0 \overset{I}{\underset{a}{\,}} 0 \ldots 0 \overset{I}{\underset{a}{\,}} 0\right) \qquad \text{Collection}$$

One further point is worth noting. A specification of the common feature or features involved in the construction of a plurality or a collection is tantamount to an intensional definition. But that does not mean that either plurality or collection is synonymous with "class." The concept of class is bounded, not experientially but logically, by the condition that it must comprise all items, and only those, that fit the criteria of the intensional definition. Conceptually, "all" is constructed by eliminating any experiential boundary that is not explicitly excepted. Plurality is not concerned with boundaries one way or the other. Collection, on the other hand, must be experientially bounded in that it requires what we might call "togetherness in experience." Thus, we may derive the intensional definition of a class from a plurality, and we may then turn the plurality into a collection by constructing a boundary around it. That collection, however, cannot be considered the embodiment of a class unless we have reason to believe that beyond its boundary there is no item that would fit the definition.

If the same operation of empirical abstraction that we hypothesized in the construction of unitary items is applied to the sensory-motor items that have been conceived as a collection, this yields a somewhat more advanced conceptual structure that I called *lot* (von Glasersfeld, Note 2). We may represent lots in the same way as collections, except for the fact that since we are now concerned with "wholeness" and "sameness" of the items rather than with their specific sensory-motor content, we again write the general "n" for the specific "a":

5) $$\left(0 \overset{I}{\underset{n}{\,}} 0 \quad 0 \overset{I}{\underset{n}{\,}} 0 \ldots 0 \overset{I}{\underset{n}{\,}} 0\right) \qquad \text{Lot}$$

It has to be emphasized that the lot-structure, like all the structures we have discussed so far, does not have the conceptual "mobility" (Piaget's term) that might make it available in the absence of perception or representation of sensory-motor material. That is to say, its construction is still dependent on the experience of a sensory-motor situation, and it therefore cannot be considered either a fully abstract concept or a numerical one.

Removal from the sensory-motor level requires what Piaget has called "reflective abstraction," that is, in our terms, the focusing of attention not on sensory-motor signals but on the results or products of prior attentional operations. Something that has been constructed by means of an attentional pattern is now reprocessed and used as raw material for a new sequence of focused and unfocused pulses. In the case of the unitary items, this creates an abstract, or *arithmetic unit*, that, in our view, represents Piaget's "element stripped of its qualities" (cf. the above quotation from Piaget, 1970). The reprocessing of a unitary item does two things: It separates the attentional pattern (that created the unity) from whatever sensory-motor material it contained and focuses an attentional pulse on it. In doing so, it creates a new unit that is again bounded by unfocused pulses. This derivation can be explicitly represented by the following:

$$0\;\overset{I}{\overline{\left(0\;\underset{n}{\overset{I}{}}\;0\right)}}\;0$$

This graphic arrangement indicates that a complete unit item constitutes the focus of the central attentional pulse. For the sake of typographical simplicity, however, I prefer to write the following:

6) 0 (0 I 0) 0 Arithmetic Unit

Though the arithmetic unit obviously belongs to the family of "wholes," or "unities," it seems unlikely that a child would construct it as the first product immediately after the acquisition of unitary items. Rather, it probably arises as a by-product of reflective abstraction, after the process of attentional iteration, which is logically the next step, has been applied to collections and lots.

When material that is reprocessed is not a single unitary item but a lot, the application of attentional pulses to the existing attentional pattern yields an iterative alternation of focused and unfocused pulses. The product of this iteration is not yet wholly abstract, because it is still experientially bounded in that it derives its boundedness from the collection that is being reprocessed. Internally, however, it consists of abstract units. I call this structure an *arithmetic lot*. Its derivation could be explicitly represented by the following:

$$\left(0\;\;\overset{I}{(0I0)}\;\;0\;\;\overset{I}{(0I0)}\;\;0\;\;\ldots\;\;0\;\;\overset{I}{(0I0)}\;\;0\right)$$

But here, again, I prefer to write the following:

7) (0 I 0 I 0 . . . I 0) Arithmetic Lot

The attentional iteration that produced arithmetic lots is the operational basis for the construction of *number*. Arithmetic lots, indeed, have numerosity—but further operations are required to specify the numerosity of a given arithmetic lot. These further operations underlie the various manifestations of "counting" and involve properties and capabilities that, in themselves, are not numerical: (a) properties of the system of number-names and the way in which it is acquired and applied; (b) perceptual processes in the area of pattern recognition, some of which have been subsumed under the term *subitizing* (i.e., the perception of small lots); and (c) particular capabilities the nervous system seems to have for the processing of elementary rhythms, especially in the kind of coordination that generates the complex results that traditionally have been covered by the term *one-to-one correspondence*. These areas will be the subjects of future investigations.

The exposition of the role of attentional patterns must, however, be extended by one further step that is decisive in the acquisition of arithmetic skills. Reflective abstraction can create a yet higher order of conceptual structure. Any arithmetic lot can be taken as material for the application of the attentional pattern that constitutes a unity. The result of such an operation corresponds exactly to what might be called a *unity of units* or a *whole number*. In our explicit representation, that would look like this:

$$0 \overset{\mathrm{I}}{(0\mathrm{I}0\mathrm{I}0 \ldots \mathrm{I}0)} 0$$

or, simplified,

8) 0 (0 I 0 I 0 . . . I 0) 0 Number

Each number, thus, can be conceived as an arithmetic unit regardless of the numerosity of the lot it contains. In the practice of arithmetic operations, of course, keeping track of specific numerosities will always be necessary. That, apart from typographical simplicity, is the reason why I have chosen the particular notation: It allows us to indicate the differences of numerosity that must be registered if the numerical concepts are to be used in any type of computation. If we compare the graphic representation of the arithmetic unit with that of, say, the number "three," we realize that they have the same unitary conceptual structure and differ only in the extent of attentional iteration they contain:

0 (0 I 0) 0	"One"
0 (0 I 0 I 0 I 0) 0	"Three"

To conclude, I should like to stress two points that, to me, are the central ideas explicated by this model. First, when we speak of "things," "wholes," "units," and "singulars," on the one hand and of "plurals," "pluralities," "collections," and "lots," on the other, we refer to conceptual structures that are dependent on material supplied by sensory experience. Insofar as these concepts involve sensory-motor signals, they do not belong to the realm of number. They enter that rarified realm through the process of reflective abstraction, which extricates attentional patterns from instantiations in sensory-motor experience and thus produces numerical concepts that are stripped of all sensory properties.

Table 1

Schematic Summary of Steps in the
Construction of the Abstract Concepts of Unit and Number

Sensory-Motor Item	$0\,\underset{a}{I}\,\underset{b}{I}\cdots\underset{n}{I}\,0$		Sequence of attentional pulses focused on sensory signals, bounded by unfocused pulses
(minimal representation)	$0\,\underset{a}{I}\,0$		Sensory-motor item characterized by specific sensory signal
Plurality		$\cdots 0\,\underset{a}{I}\,0\;0\,\underset{a}{I}\,0\;0\,\underset{a}{I}\,0 \cdots$	Open-ended sequence of sensory-motor items having a common sensory characteristic
Collection		$(0\,\underset{a}{I}\,0\;0\,\underset{a}{I}\,0 \cdots 0\,\underset{a}{I}\,0)$	Experientially bounded plurality
1st (empirical) abstraction Unitary Item	$0\,\underset{n}{I}\,0$		Abstracted "figurative" pattern of sensory-motor item, still dependent on sensory material
Lot		$(0\,\underset{n}{I}\,0\;0\,\underset{n}{I}\,0 \cdots 0\,\underset{n}{I}\,0)$	Abstracted "figurative" pattern of collection, still dependent on sensory material
Arithmetic Lot		$(0\;I\;0\;I\;0 \cdots I\;0)$	Sequence of unitary items reduced to iterative attentional pattern
2nd (reflective) abstraction Arithmetic Unit	$0\;\underset{(0\;I\;0)}{I}\;0$		Attentional pattern of unitary item reprocessed to constitute focal item of unitary pattern
Number		$0\;\underset{(0\;I\;0\;I\;0 \cdots I\;0)}{I}\;0$	Attentional pattern of arithmetic lot reprocessed to constitute focal item of unitary pattern

Second, we may generalize and say that, conceptually, all whole numbers are indeed unitary wholes and, as Euclid (Book VII, 1926) correctly saw, they are themselves composed of units; but we can now add, on the basis of our theoretical model, that whereas the component units are the non-numerical raw material of focused attentional pulses, the wholeness of the number concepts is made of different stuff—it derives from the *structural pattern* in which these pulses are arranged.

It now remains to be seen whether this model provides a new and more successful approach to an understanding of the still problematic activities of counting and the operations involved in establishing specific numerosities.

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