

BIO-LOGIC

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If in the past somebody had asked about the fundamental difference between the animate and inanimate world, the most likely answer would have been: living organisms possess that unique feature of reproduction, the capacity of self-replication, while dead matter obviously does not have this power. Today, however, I would be hesitant to give this answer. We know by now of several schemes, even very simple schemes, which display the features of self-replication (Von Neumann, 1951; Lofgren, 1958) and in the course of this symposium you will be exposed to some interesting ideas on error correction (Lofgren, 1962) which is, in a sense, a more general case of self-reproduction. But I doubt whether we would classify systems displaying these features as "living organisms." I certainly would not do so, because I believe that this capacity of self-replication is only a particular manifestation of a much more general principle which is the basis for all life. I shall come to it in a moment, after I have shown in a "Gedanken Experiment" that self-replication is not fundamental to life. Assume that I have a gadget which, by the flick of a switch, inhibits all meioses on this globe. As you may remember, meiosis is the chromosome-pair-splitting cell division which initiates oogenesis or spermatogenesis; hence, without meiosis, no sexual reproduction. Imagine that I turn on this switch. Certainly no drastic change will occur. We may happily live on as before, although after a while we may observe fewer and fewer people around us. This side effect, however, could be counteracted by another gadget which would secure immortality. Since this gadget is not yet on the market, let me turn off the switch again, and let us look for a moment on this peculiar fertility-mortality interdependence, which is a consequence of one of the neat tricks of life, namely, to live on life. Since the so-called "higher forms" live on "lower forms," they must be able to break up the lower forms into those constituents out of which the higher forms are built. If these lower forms were so stable that they could not be decomposed, higher forms could not develop. In other words, the ultimate in stability-immortality-leads to stagnation and sets the end point in evolution. Since for each level of stability there always developed a higher level which could crack up the lower ones, fertility is the way out of this problem. It seems to me, however, that those who have their fingers on the bomb triggers have a different solution for this problem in mind.

As we have seen, the capacity of self-reproduction, although omnipresent in the animate world, is tied up with a host of other characteristics of living organisms and cannot claim to be a fundamental principle.

I propose to consider as a fundamental principle of living things their capacity to form coalitions. Thus, sexual reproduction appears to be only a particular manifestation of this capacity. What I call a "coalition" is an aggregate of elements which jointly can do things which all of them separately could never achieve. In more precise terms this means that a coalition is characterized by a superadditive nonlinear composition rule where some measure Φ of the whole is more than the sum of the measure of the parts:

$$\Phi(x, y) > \Phi(x) + \Phi(y)$$

It is noteworthy that as an example of such a measure negentropy ($-H$) can be cited, because we have with Shannon (1949)

$$H(x, y) \leq H(x) + H(y)$$

with equality only if x and y are completely independent. This suggests that information plays an important part in the formation of coalitions and that information reduction is one of the desired outcomes.

Let me show two examples that aggregations in the inanimate world increase the instability of the aggregate and thus exemplify a subadditive nonlinear composition rule, while aggregates of living elements secure higher survival values (stability) for the individual elements and thus are capable of forming true coalitions.

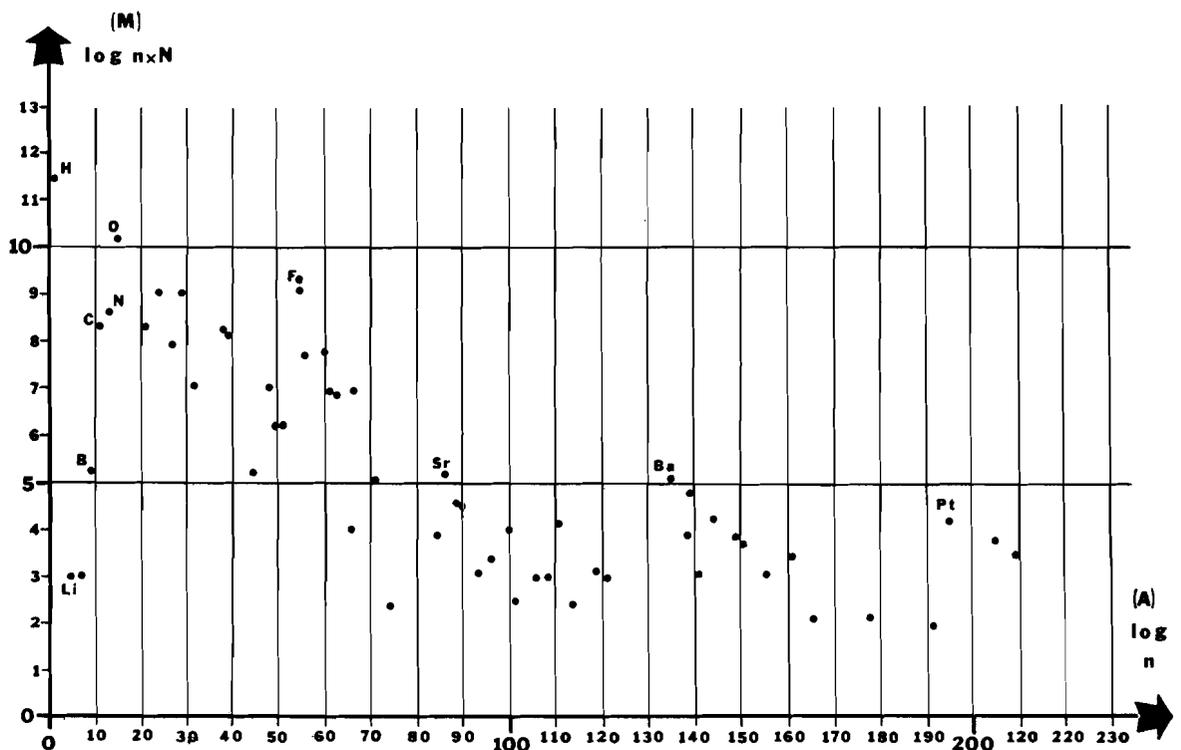


Fig. 1. Relative abundance of elementary particles forming the various chemical elements.

Figure 1 is a representation of some measure of the relative abundance of the chemical elements (adapted from Grotrian, 1934) in the universe ordered according to their atomic weights, which represent a fair measure of the number, n , of fundamental particles aggregated in the atomic nuclei of the various elements. Note that the abscissa does not represent just N , the number of atoms of a certain kind found in an arbitrarily large unit of cosmic volume,* but the number of fundamental particles $n \cdot N$ which constitute the various elements. In other words, Fig. 1 gives an indication of the probability of finding an elementary particle as a member of a nucleus of one of the various elements. From this figure it is easily seen that if a cosmonaut were to make a random collection of elementary particles, he would find with overwhelming probability particles forming the nucleus of hydrogen, i.e., single particles. Five percent of his catch would be particles found in nuclear aggregates of 16 particles (oxygen) and 0.5% in aggregates of 56 (iron), the third most probable aggregate. The probability

*This volume has been chosen to give the number of fundamental particles which constitute oxygen, a value of about 10^{10} .

of finding elementary particles in higher aggregates becomes extremely small, being always less than 10^{-7} for aggregates of more than 150 particles. Clearly, the "stability per elementary constituent" is decreased in larger and larger aggregates, which suggests a subadditive composition rule.

Applying similar reasoning to living organisms, our first guess might be that the situation is about the same, because small cellular aggregates, e.g., mosquitoes, are by far more abundant than large ones, e.g., elephants. True, but this doesn't say anything about the stability per elementary constituent. In order to give a rough estimate of this stability, in exact analogy to our previous example, I have compiled in Fig. 2 the global abundance of cells forming the various species ordered according to the number, n , of cells aggregated in typical specimen of the various species. Note again that the abscissa does not represent just N , the number of specimens, i.e., the global population of a certain species, but the number of cells $n \cdot N$ which constitute the various species. In other words, Fig. 2 gives an indication of the probability of finding a cell as a member of cellular aggregates of various sizes. From this figure it is easily seen that if a statistical cytologist were to make a random collection of cells, he would find with overwhelming probability mammalian cells, and amongst these, human cells in greatest abundance. It may come as a surprise, but the number of cells belonging to the participants of this symposium outnumber by a few orders of magnitude all the unicellular organisms on this globe. This sug-

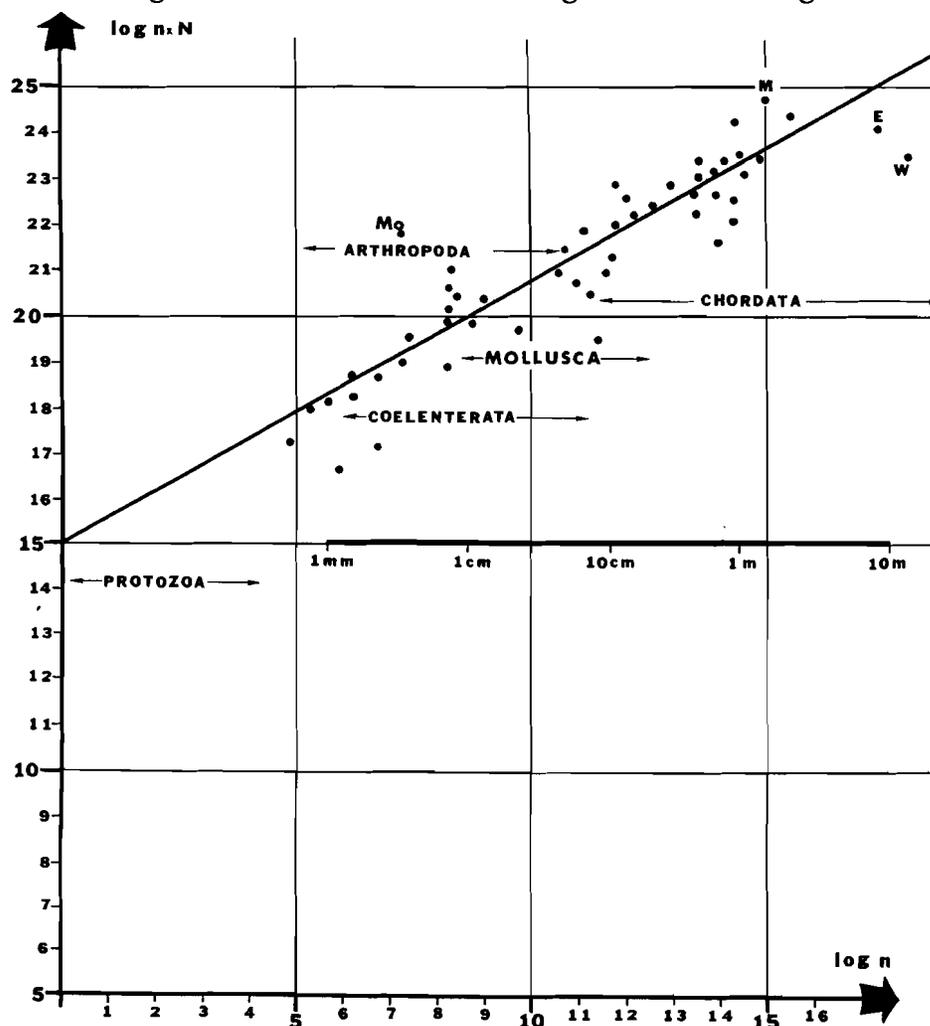


Fig. 2. Global abundance of cells forming the various species (E, elephant; M, man; Mo, mosquito; W, whale).

gests that the stability of elementary constituents of living organisms is very much increased if these constituents form aggregates of higher and higher complexity. Here we have a case of a superadditive composition rule—a true coalition—where the pay-off comes in the form of an increased survival value for the elementary constituents, the cells, whenever they aggregate to form systems of higher organization.

I hope that these introductory examples have made it sufficiently clear that if I am to elaborate my theme, "bio-logic," I have to concern myself with the logic of coalitions.

We may now ask, what is indeed this logic of coalition which makes an aggregation superadditive? Let me illustrate this situation first with a simple example which I shall generalize in a moment. Assume that there is a highly specialized zoologist, Z , whose only knowledge is the truth of the proposition x_1 :

$$x_1 = \text{"elephants are gray"}$$

Assume, furthermore, that there is a highly specialized physicist, P , whose only knowledge is the truth of the proposition x_2 :

$$x_2 = \text{"electrons are negative"}$$

The "truth table" of the two scientists Z , P with respect to the two propositions x_1 , x_2 is clearly

| x_1 | x_2 | Z | P | |
|-------|-------|-----|-----|-----------|
| 0 | 0 | 0 | 0 | |
| 0 | 1 | 0 | 1 | 0 = false |
| 1 | 0 | 1 | 0 | 1 = true |
| 1 | 1 | 1 | 1 | |

because the zoologist describes his universe with the logical function

$$x_1 \cdot (x_2 \vee \bar{x}_2)$$

or, in words,

"elephants are gray, and electrons are negative or electrons are not negative"

while the physicist does not care about the color of elephants and describes his universe by the logical function

$$x_2 \cdot (x_1 \vee \bar{x}_1)$$

Following Carnap, Bar-Hillel (1955), and others, I shall introduce the "strength," S , of a logical statement, which is defined by the number of ways in which such a statement can be false, $N(0)$, divided by the number of ways it can be either true or false, $N(1,0)$. Since with m propositions we must have

$$N(1,0) = 2^m$$

the strength of a logical statement is given by

$$S = N(0) \cdot 2^{-m}$$

with the two extreme values

$$S = 0 \text{ for tautology}$$

and

$$S = 1 \text{ for contradiction}$$

Although it appears at first absurd to call a statement stronger than another one if it is false in more instances, on second thought the definition given above follows our intuition insofar as we would consider a statement very weak if it is almost always true, independent of whether the propositions it contains are true or false. Such a statement would tell us very little about the universe, as is seen in the case of the tautology which is always true but says nothing ($S = 0$). Let us apply this measure of logical strength to the wisdom of our two scientists. We find that in both cases the strength of their statements is $\frac{1}{2}$. However, if the two scientists form a coalition by establishing, say, a Biophysical Society, the society is ambivalent with respect to neither the color of elephants nor the charge of electrons and its truth table would read:

| x_1 | x_2 | $Z \& P$ |
|-------|-------|----------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

with the logical strength of the wisdom of the society being $\frac{3}{4}$, which indeed represents an appreciable gain over the previous situation, where the two scientists were sitting isolated in their ivory towers.

The mechanics of computing truth tables of coalitions from the truth tables of individual partners is quite obvious: the product of the truth values in each row represents the truth value of the coalition in this row. Thus, the presence of a zero in such a row will convert all ones into zeros and the resulting number of ways in which false statements can be made has increased.

It is easy to generalize this process for an arbitrary number of k elements E_1, E_2, \dots, E_k , dealing with an arbitrary number of m propositions x_1, x_2, \dots, x_m , each of these elements E_i describing its knowledge in form of a logical function $F_i(x_1, x_2, \dots, x_m)$ with strength S_i . Let two of these elements, say E_1, E_2 , form a coalition. We may ask the following question: What is the most probable resulting strength $S_{1,2}$ of this coalition? I shall forego cumbersome arithmetic and give you immediately an approximate result. It turns out that with overwhelming probability the resulting strength will be

$$S_{1,2} = S_1 + S_2 - S_1 S_2$$

or, even simpler, if $W = 1 - S$ defines the "logical weakness" of a statement, the resulting weakness will be

$$W_{1,2} = W_1 W_2$$

In other words, the logical weakness of a coalition is with overwhelming probability the product of the logical weaknesses of its components. If, for simplicity, we assume all elements equally ignorant

$$W_1 = W_2 = W_3 = \dots = W_0 < 1$$

the coalition weakness of k elements decreases with the k th power of the weakness of the individual element:

$$W_k = W_0^k$$

and thus the coalition strength increases according to

$$S_k = 1 - e^{-\alpha k}$$

where

$$\alpha = \ln \frac{1}{(1 - S_0)} > 0$$

This expression for S_k , which indicates an asymptotic increase of the measure of knowledge for a coalition which keeps growing by absorbing more and more members, allows many different interpretations. In the middle of the last century it would have been interpreted as our hopeless strife for absolute truth, which we may never reach, however close we may come. The second half of the last century would have observed that, since there is only a finite number of propositions, m , in order to be in possession of the perfect truth it is only necessary that

$$S_k = 1 - 2^{-m}$$

This would insure that the final truth table contains only a single "true" precisely for that combination of the truth values of the propositions which represents a unique and unambiguous description of the universe. Indeed, some of the great physicists of that period believed that all essential problems had been solved and what was left for the epigones was only to clean up a few side issues. However, the Curies, Planck, and Einstein, discovered not only that some of the "side issues" were the main issues, but also that there are plenty of new issues. Thus, the beginning of our century saw the fallacy of our progenitors in their trust in a fixed number of m propositions. This number constantly grows with new discoveries which add new variables to our system of knowledge. In this connection it may amuse you that in order just to keep the logical strength of our wisdom from slipping, the ratio of the rate of coalescing, k , to the rate of discovery, m , must obey the inequality

$$\frac{\dot{k}}{m} \geq k \cdot \ln 2$$

I have the feeling that today, with our tremendous increase in experimental techniques, \dot{m} is occasionally so large that the above inequality is not fulfilled, and we are left with more riddles than before.

To this frustration to reach perfect truth we, children of the second half of the twentieth century, have added another doubt. This is the suspicion that noise may enter the most effective coalition, flipping an established "false" into a deceptive "true" or, what might be even worse, flipping an irrelevant "true" into an unwarranted "false."

The occurrence of such spontaneous errors is far from an uncommon event. Conservative estimates suggest about 10^{14} elementary operations per second in a single human brain. If we can believe the recent work of Hyden (1960) and Pauling (1961), these operations are performed on about 10^{21} molecules. From stability considerations (Von Foerster, 1948) we may estimate that per second from 10^9 to 10^{11} molecules will spontaneously change their quantum state as a result of the tunnel effect. This suggests that from 10^{-3} to $10^{-1}\%$ of all operations in the brain are afflicted with an intrinsic noise figure which has to be taken care of in one way or another. I am happy to see that later on in this symposium issue will be taken with this intriguing problem (Lofgren, 1962).

I shall now leave this short historical interlude and turn to possible realizations of superadditive composition rules on the most elementary level of our information processing network, namely, on the neurons themselves.

In Fig. 3 I have collected a few examples of neurons to which the endbulbs of preceding neurons A, B, C, . . . are attached (column I). We know from the work by McCulloch (1958) and others (Blum, 1960; Verbeek, 1960) that if the

| | I | II | III | IV |
|---|---|--------------------------------------|---|---|
| | SCHEME | ϕ | F | S |
| 1 |  | 0 1 2 | \mathcal{T} A \mathcal{C} | 0. 0.5 1.0 |
| 2 |  | 0 1 2 3 4 | \mathcal{T} A v B B A · B \mathcal{C} | 0. 0.25 0.50 0.75 1.00 |
| 3 |  | 0 1 2 3 4 5 6 7 | \mathcal{T} A v B v C B v C A · B v C A · C v B · C B · C A · B · C \mathcal{C} | 0. 0.125 0.250 0.375 0.625 0.750 0.875 1.000 |
| 4 |  | 0 1 2 3 4 5 6 7 | \mathcal{T} A v B v C A v B v C A · B v A · C v B · C A · B v A · C v B · C A · B · C A · B · C \mathcal{C} | 0. 0.125 0.125 0.500 0.500 0.875 0.875 1.000 |
| 5 |  | | $p(N_i) = \frac{N! Z^{-N}}{\prod N_i!}$ $\sum N_i = N \dots N_i = N/n \quad p = \max$ $\sum y_i N_i = \phi \dots N_i = A e^{-\beta y_i} \quad p = \max$ | |

Fig. 3. Neural functionals defined by various synaptic distributions.

threshold Θ of these neurons increases in unit steps, in most cases the neuron will compute a different logical function with the input neurons A, B, C, . . . as variables. For each of the different connection schemes, 3.1 to 3.4 in column I, the logical function computed is given in column III if Θ assumes values as listed in column II. \mathcal{T} stands for tautology, \mathcal{C} for contradiction. Column IV gives the logical strength s for the functions computed. There are three points which I would like to make in connection with this figure, points which I believe are pertinent to my theme.

First, I would like to draw your attention to the important observation made by McCulloch and Ashby (1945) that a given synaptic pattern in connection with a variable threshold represents a functional in the sense that the synaptic pattern

defines a set Σ of functions F_i in such a manner that to each F_i in Σ corresponds an F_j in Σ , namely, the one with the next higher logical strength, the correspondence being established by the unit shift of the operator Θ .

Second, it is worthwhile to note that for the same number of input neurons a variety of sets of logical functions may be computed, depending upon the distribution of endbulbs over the input neurons. This is exemplified in schemes 3.3 and 3.4, which both have three input neurons and six endbulbs, with the endbulbs distributed (1,2,3) in the first case and (2,2,2) in the second case. In general, it is easy to see that N endbulbs distributed over n input neurons will permit the formation of $p(N,n)$ different functionals, where $p(N,n)$ gives the number of partitions of N elements into precisely n parts. At this point it may also be interesting to note that while the endbulb distribution (1,2,3) produces—omitting tautology and contradiction—a functional of six "useful" logical functions with great sensitivity to changes in threshold, the endbulb distribution (2,2,2) produces only three useful functions, yet with reasonable stability to threshold variations. More sophisticated logical stable networks have been extensively studied by McCulloch and his school (McCulloch, 1958; Blum, 1960, 1961; Verbeek, 1960, 1961; Cowan, 1961). Nevertheless, it is tempting to pursue the relationship between the endbulb distribution and the generated set of logical functions a bit further on a statistical basis. Since we know of several neuron types with synaptic connections going into the thousands and tens of thousands, such an approach seems to be not completely unwarranted. Take n input neurons $x_1, x_2, \dots, x_i, \dots, x_n$, each having $N_1, N_2, \dots, N_i, \dots, N_n$ endbulbs connected to our neurons of interest as indicated in Fig. 3 (5). The total number of endbulbs

$$N = \sum_1^n N_i$$

is assumed to be fixed. Since each input neuron is distinguishable from any other, the number of ways in which a particular distribution of endbulbs may be obtained is

$$\frac{N!}{\prod N_i!}$$

and the number of possible distributions is

$$Z^N$$

Thus, the probability of a particular distribution is

$$p(N_i) = \frac{N! Z^{-N}}{\prod N_i!}$$

with

$$\sum_1^n N_i = N$$

This is nothing else but Boltzmann's way of counting distinguishable states for the molecules of a diluted, nondegenerate gas. Hence, the answer for the distribution which has greatest probability can just be copied from introductory textbooks in statistical thermodynamics. One obtains:

$$p(N_i) = \max \quad \text{for } N_i = N_0 = N/n$$

that is, a uniform distribution. The set of logical functions generated by this connection pattern is minimum and contains n useful logical functions with a maximum threshold safety margin of N_0 unit steps. This is also intuitively quite plausible considering the great redundancy of this connection pattern provided

by the multitude of like connectivities. However, it may be argued that each input neuron may transmit, on the average, a different amount of information, which would require the i th neuron to have ϕ_i units of energy available per unit time.

Under these circumstances it may be impossible to maintain the distribution suggested above, because the energy produced per unit time on the surface of a perikaryon is limited to, say, Φ . Hence, we have to consider an additional restriction, namely, that the total energy released per unit time should be constant

$$\sum \phi_i N_i = \Phi$$

The problem of finding the most probable distribution for our endbulbs, observing the restriction mentioned above, leads, as you may recall, to Boltzmann's energy distribution:

$$N_i = A e^{-\beta \phi_i}$$

where A and β are constants, depending upon Φ and N in a somewhat involved manner. Here the most vigorous neuron develops the least number of endbulbs, while the slowpoke is endowed with the loudest voice. The generated set of logical functions is maximum and contains of the order of N useful logical functions with a minimum threshold safety margin of the order of a single unit step.

After this little excursion into thermodynamics, I shall now turn to my third point in reference to Fig. 3, and this point may be the most pertinent one regarding the topic of my presentation. It involves column IV of Fig. 3, which lists S , the logical strength, of the functions generated by a certain connection pattern and determined by the various threshold levels.

A brief inspection of this column shows that the logical strength increases monotonously with increasing threshold. Translated into our coalition language this means that for low threshold values the n input neurons are still acting as if they were almost independent. However, with increasing threshold values, stronger and stronger coalitions between the n input neurons are formed. In other words, the output of the neuron which collects the wisdom of its n input neurons reports the wisdom of their coalition with the threshold as coalescing agent. Of course, an increasing threshold can be replaced by afferent inhibitory fibers which may be stimulated by some neurons watching the output of our coalition neuron. Increasing inhibition will thus produce two effects. First, the logical functions computed will approach closer and closer the one function which represents most closely that "what is the case," as we have seen earlier; second, the output activity of our coalition neuron will be further and further diminished until it fires only if "what is the case" is indeed the case. In other words, formation of coalitions by inhibition, and adaptation by inhibition in connection with diminished activity are two processes which go hand in hand. The only missing link in this equivalence is: What regulates the inhibition?

The following example may suggest an answer. Assume two layers of neurons where each neuron in the upper layer grows its axons toward its corresponding neuron in the lower layer. However, the intermediate tissue will cause random-walk deviations, and with the two different target sizes for excitation and inhibition, i.e., perikarya and dendritic ramifications, respectively, the response of the lower layer as a result of the random connectivity will be

$$R(r_2) = A_1 \int_0^{\infty} S(r_1) e^{-(r_1-r_2)^2/\sigma_1^2} da - A_2 \int_0^{\infty} S(r_1) e^{-(r_1-r_2)^2/\sigma_2^2} da$$

where r_1 and r_2 are the vector loci on the upper and lower layers, respectively; $S(r_1)$, $R(r_2)$ are stimulus and response; and A_1 , σ_1 and A_2 , σ_2 are amplitude and variance of excitation and inhibition, respectively. It is well known (Inselberg, et al., 1960) that such a connectivity would produce an adequate computer for second derivatives of the stimulus pattern—or a "contour detector," if you wish—if the following condition is fulfilled:

$$A_1\sigma_1^2 = A_2\sigma_2^2$$

The necessity of this condition is easy to see, because only under this condition does a uniform stimulus, $S(r_1) = \text{const}$, elicit no response in the lower layer, i.e., $R = 0$; and indeed, a uniform stimulus does not have a contour. The question arises, how can this randomly growing net accomplish the trick of obeying the above-mentioned condition? A genetic preprogramming of the growth of the fibers according to our condition can be dismissed as absurd. In addition, this suggestion would spoil our simple assumption of random growth. Here, our principle of adaptation by inhibition will come in handy. Assume that initially the connections between our layers are such that a uniform stimulus will elicit a response in the second layer, that is, the left-hand side of our condition will predominate (Fig. 4a). If this response stimulates growth in the descending axons and activates further dendritic ramifications in the target neurons in the interface between our two layers (Fig. 4b), it is the right-hand side which gains, because A_1 and A_2 will increase in proportionate amounts, σ_1 , the facilitatory variance, will remain the same, but σ_2 will increase due to the increase of the inhibitory target size (Fig. 4c). This stimulus for growth will last until inhibitions precisely counterbalance excitations, at which instant the zero-response condition is fulfilled, the growth stimulus ceases, and the net is ready for contour extraction. We may also say that the net is now "adapted" for a uniform stimulus distribution. Uniformity is not to be registered; it is the perturbation of uniformity which is to be recognized. In passing, I may mention that during the

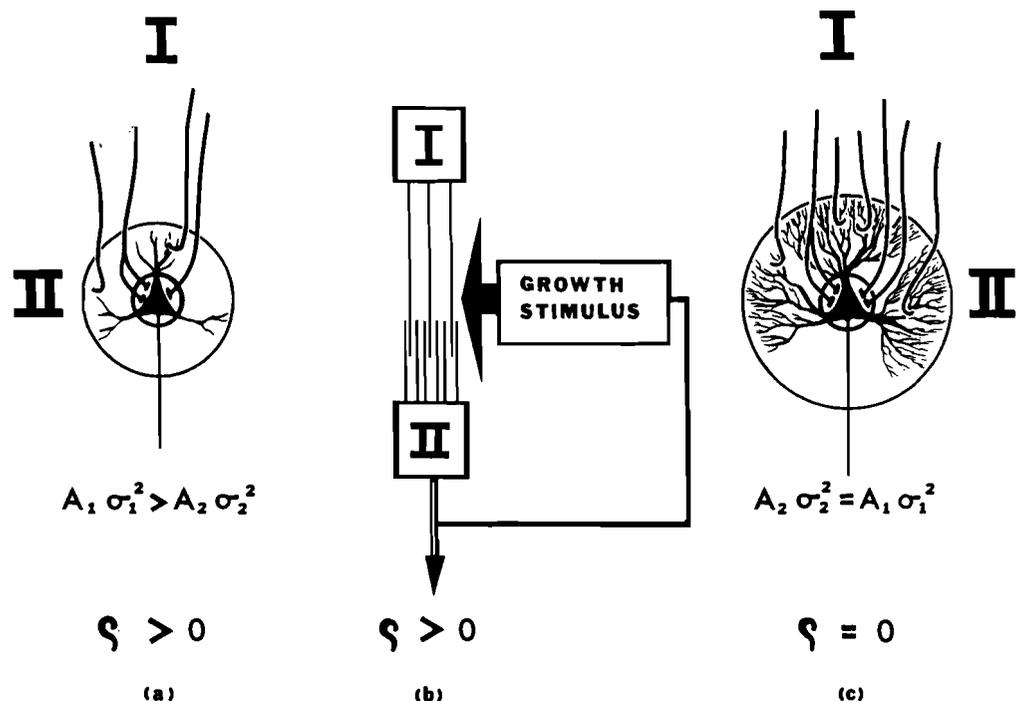


Fig. 4. Adaptation by inhibition: (a) Initial state; (b) axonal and dendritic growth; (c) final state of adaptation.

process of increasing inhibition the logical strength of each neuron in the lower layer has moved up from an initial value ϵ to a final value of $0.5 + \epsilon$, where ϵ depends inversely on the number of coalescing neurons.

Up to this point I have only spoken of the blessings of coalitions. However, I cannot conclude my remarks on the logic of coalitions without a warning. This warning refers to a peculiar instability which seems to be inherent in aggregates of reproducing, coalition-forming elements. Consider N elements and their rate of reproduction \dot{N} . At any instant of time, this rate will be proportional to the number of elements:

$$\dot{N} = \alpha(N) \cdot N$$

where the proportionality $\alpha(N)$ will depend on the number of elements and the energy supply available for each element in such a manner that for elements competing for these commodities, $\alpha(N)$ will decrease as the number of competitors increases, while if the elements are capable of forming ever-stronger coalitions, $\alpha(N)$ will increase with the number of elements. If, as representatives of these two kinds of elements, we chose fruit flies living in a container in the first case, and human beings inhabiting this globe in the second case, the following expressions for $\alpha(N)$

$$(I) \quad \alpha(N) = a - bN$$

$$(II) \quad \alpha(N) = cN^{1/k}$$

where a , b , c , and k are constants, give very good agreement with observations (Vance, 1959; Von Foerster et al., 1960). Solving our differential equation in \dot{N} for N , while observing the different suggestions for $\alpha(N)$, one obtains for the two cases:

$$(I) \quad N = \frac{ae^{at}}{b(e^{at} - 1) + a/N_0}$$

$$(II) \quad N = \frac{K}{(t_0 - t)^k}$$

While a competitive population will asymptotically reach an equilibrium when the number of elements is $N_\infty = a/b$, a coalescing population exhibits an instability at $t = t_0$, at which instant the number of elements approaches infinity. In case you would like to know the date at which for the human race this instability occurs: it will be on Friday, November 13, 2026. It is up to the bionicists to improve the bio-logic of our human coalition, which has failed us in a decisive point: it has not given us an inhibitory mechanism, a "peoplostat," so to say, which would keep our presently ever-faster-growing population on a constant level. If we go on as we do now, we shall end like lemmings. Let us not give in so easily. Should we, who have wrung such decided advantages from many a shortcoming, be defeated by our own nature? We must do at least as well as fruit flies!

We have strong support. When we leave this conference we shall do so with greatly increased knowledge and wisdom, thanks to our enlightened organizers, who enabled us to participate in a most promising coalition, the Second Symposium on Bionics.

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